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Parametrically Adaptive Wavenumber Processing for Mode Tracking in a Shallow Ocean Experiment

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***Abstract*—The shallow ocean is a dynamic environment requiring an adaptive processor. Parametrically adaptive processing implies embedding a parametric process model enabling a joint sequential processor capable of tracking oceanic variations. Here we address the problem of estimating or tracking modal functions in the ocean while jointly adjusting (adaptively) the inherent normal-mode propagation model parameters (wavenumbers) based on the data available from the Hudson Canyon experiment.**

***Index Terms*—adaptive model-based processor, sequential Bayesian processor, sequential Monte Carlo, particle filter, unscented Kalman filter**

I. INTRODUCTION

The shallow ocean is an ever-changing dynamic environment requiring a processor able to adapt to changes in a consistent manner. Parametrically adaptive processing implies embedding a process model coupled to a parametric evolution model enabling a joint processor that can be achieved using a recursively or equivalently sequentially. Sequential processing enables the realization of such a processor in order to account for changes especially in a shallow ocean environment. The processor tracks these variations by adjusting the embedded parameters that are capable of capturing the environmental changes (nonstationary spatial/temporal variations).

The shallow ocean is a particularly challenging signal processing environment primarily because of its inherent dynamics created by temperature variations in the upper layers and both internal and external disturbances that directly alter the sound propagation throughout. Temperature variations directly impact sound speed due their strong interrelationship, while internal disturbances can be related to fish sounds (snapping shrimp, mammal communications). External disturbances are directly related to wind induced wave motion, shipping noise and other surface related noises. In all, the shallow ocean is quite a hostile environment to attempt to extract meaningful information from directly without sophisticated processing techniques.

Bayesian sequential processing incorporating propagation models along with their inherent environmental parameters as well as measurement and noise models offers a robust, parametrically adaptive solution to signal processing problems in this nonstationary environment. Sequential Bayesian techniques enable a class of processors capable of performing in an uncertain, nonstationary (varying statistics), non-Gaussian, variable shallow ocean environment. Here we address the problem of estimating or tracking modal functions in a hostile shallow ocean while jointly adjusting (adaptively) the inherent normal-mode propagation model parameters (wavenumbers).

Previous work on this problem has investigated adaptive solutions under Gaussian assumptions using approximate non-linear processors such as extended Kalman filters with some success; however, here we attack the problem with the sequential Bayesian construct enabling the joint modal function and wavenumber estimation to proceed without any limiting statistical assumptions. It has already been shown that this parametrically adaptive approach can operate successfully in this environment [25], [26] when estimating modal coefficients; however, here we construct processors (as before [27]) to adapt to the horizontal wavenumber—a more environmentally sensitive parameter.

We begin by briefly reviewing the formulation of the problem in a state-space framework and developing the necessary mathematics for processor design (more details can be found in [27]. After proceeding through the design using simulated data, we apply the adaptive processor to the Hudson Canyon experimental data. The performance of the Bayesian processor is analyzed indicating its capability to track both modes and wavenumber simultaneously while jointly enhancing the raw hydrophone measurements.

The basic approach we employ to solve this problem is Bayesian *model-based*. Incorporating a propagation model into a signal processing scheme has evolved over a long period of time where it was recognized that by embedding a physics-based representation can significantly improve the processing [1]-[5]. In ocean acoustics there are many problems of interest [6]-[14] governed by propagation models of varying degrees of sophistication.

In this paper, we are primarily interested in investigating the performance of the unscented Kalman filter (*UKF*) and the particle filter (*PF*) with the objective of analyzing their performance on pressure-field data from the well-known Hudson Canyon experiments performed on the New Jersey shelf [11], [12]. The *PF* is a sequential Markov chain Monte Carlo (*MCMC*) Bayesian processor capable of providing reasonable performance for a multi-modal problem estimating a non-parametric representation of the posterior distribution [24]. The *UKF* is a unimodal processor capable of representing any single peaked distribution using a statistical linearization technique based on sigma points that deterministically characterize the posterior [24]. Here we compare their performance on the raw Hudson Canyon data.

The state-space representation of our problem is briefly presented in Section II leading to the formulation of the forward propagator. The particular algorithms employed were discussed previously [27]. The design of the Bayesian processor for this shallow oceanic problem is discussed in Section III and the results are given in Sec. IV.

II. STATE-SPACE PROPAGATOR

For our ocean acoustic signal enhancement problem we assume a horizontally-stratified ocean of depth h with a *known* horizontal source range r_s and depth z_s and that the acoustic energy from a point source can be modeled as a trapped wave governed by the Helmholtz equation [9], [13]. The standard separation of variables technique and removing the time dependence leads to a set of ordinary differential equations, that is, we obtain a "depth only" representation of the wave equation which is an eigenvalue equation in z with

$$\frac{d^2}{dz^2}\phi_m(z) + \kappa_z^2(m)\phi_m(z) = 0,\ m = 1, \cdots, M \tag{1}$$

whose eigensolutions $\{\phi_m(z)\}$ are the so called *modal functions* and κ_z is the wavenumber in the z-direction. These solutions depend on the sound speed profile, $c(z)$, and the boundary conditions at the surface and bottom as well as the corresponding *dispersion* relation given by

$$\kappa^2 = \frac{\omega^2}{c^2(z)} = \kappa_r^2(m) + \kappa_z^2(m), \qquad m = 1, \ldots, M \tag{2}$$

where $\kappa_r(m)$ is the horizontal wavenumber associated with the m-th mode in the r direction and ω is the harmonic source frequency.

By assuming a known horizontal source range *a priori*, we obtain a range solution given by the Hankel function, $H_0(\kappa_r r_s)$ enabling the pressure-field to be represented by

$$p(r_s, z) = \sum_{m=1}^{M} \beta_m(r_s, z_s)\phi_m(z) \tag{3}$$

where p is the acoustic pressure; ϕ_m is the m^{th} modal function with the modal coefficient defined by

$$\beta_m(r_s, z_s) := q\ H_0(\kappa_r r_s)\ \phi_m(z_s) \tag{4}$$

for q is the source amplitude and H_0 is the zero-th Hankel function at horizontal wavenumber and source range r_s.

The depth-only eigen-equation can easily be transformed to *state-space* form by defining the state vector of the m-th mode as $\phi_m(z) = [\phi_m(z) \ \ \frac{d}{dz}\phi_m(z)] := [\phi_{m1}(z) \ \ \phi_{m2}(z)]^T$. Assuming that the ocean acoustic noise can be characterized by additive uncertainties, we can extend the deterministic state equation for the M-modes, that is, $\Phi(z) := [\phi_1(z)| \cdots |\phi_M(z)]^T$ leading to the following $2M$-dimensional Gauss-Markov representation of the model:

$$\frac{d}{dz}\phi(z) = \mathbf{A}(z)\phi(z) + \mathbf{w}(z) \tag{5}$$

where $\mathbf{A} := \mathrm{diag}[\mathbf{A}_1 \ \ \cdots \ \ \mathbf{A}_M]$ and $\mathbf{w}(z) = [w_1 \ \ w_2 \ \ \ldots \ \ w_{2M}]^T$ is additive, zero-mean random noise. The overall state vector is

$$\phi(z) = [\phi_{11} \ \ \phi_{12} \mid \phi_{21} \ \ \phi_{22} \mid \ \ \ldots \ \ \mid \phi_{M1} \ \ \phi_{M2}]^T \tag{6}$$

This representation leads to the $measurement$ equations that we can write as

$$p(r_s, z) = \mathbf{C}^T(r_s, z_s)\phi(z) + v(z) \tag{7}$$

where

$$\mathbf{C}^T(r_s, z_s) = [\beta_1(r_s, z_s) \ \ 0 \mid \ \ \cdots \ \ \mid \beta_M(r_s, z_s) \ \ 0] \tag{8}$$

The random noise terms $\mathbf{w}(z)$ and $v(z)$ can be assumed Gaussian and zero-mean with respective covariance matrices, $\mathbf{R}_{ww}$ and $\mathbf{R}_{vv}$. The measurement noise ($v(z)$) can be used to represent the "lumped" effects of near-field acoustic noise field, flow noise on the hydrophone and electronic noise. The modal noise ($\mathbf{w}(z)$) can be used to represent the "lumped" uncertainty of sound speed errors, distant shipping noise, errors in the boundary conditions, sea state effects and ocean inhomogeneities that propagate through the ocean acoustic system dynamics (normal-mode model). These assumptions, with known model parameters lead to the optimal solution of the state estimation problem [18].

Since our array spatially samples the pressure-field discretizing depth, we choose to discretize the differential state equations using a central difference approach for improved numerical stability [27] which leads to the following set of difference equations for the m-th mode for $\triangle z_\ell := z_\ell - z_{\ell-1}$

$$\begin{aligned} \phi_{m1}(z_\ell) &= \phi_{m2}(z_{\ell-1}) \\ \phi_{m2}(z_\ell) &= -\phi_{m1}(z_{\ell-1}) + \Big(2 - \triangle z_\ell^2 \kappa_z^2(m)\Big)\phi_{m2}(z_{\ell-1}) \end{aligned} \tag{9}$$

with each of the corresponding modal A-submatrices given by

$$\mathbf{A}_m(z) = \begin{bmatrix} 0 & 1 \\ -1 & 2 - \triangle z_\ell^2 \kappa_z^2(m) \end{bmatrix}; \quad m = 1, \cdots, M \tag{10}$$

The "parametrically adaptive" processor evolves from this representation by defining a parameter set of the horizontal wavenumbers to vary as before [27]. We define the *parameter vector* as $\theta_m(z) := \kappa_r(m);\ m = 1, \cdots, M$ and a new "augmented" state vector as $\Phi_m(z_\ell; \theta_m) := \Phi_m(z_\ell) = [\phi_{m1}(z_\ell)\ \phi_{m2}(z_\ell) \mid \theta_m(z_\ell)]^T$.

With this choice of parameters (horizontal wavenumber) the augmented state equations for the m-th mode become

$$
\begin{aligned}
\phi_{m1}(z_\ell) &= \phi_{m2}(z_{\ell-1}) + w_{m1}(z_{\ell-1}) \\
\phi_{m2}(z_\ell) &= -\phi_{m1}(z_{\ell-1}) + \Big(2 - \triangle z_\ell^2 \big(\frac{\omega^2}{c^2(z_\ell)} - \theta_m^2(z_{\ell-1})\big)\Big) \\
&\times \;\; \phi_{m2}(z_{\ell-1}) + w_{m2}(z_{\ell-1}) \\
\theta_m(z_\ell) &= \theta_m(z_{\ell-1}) + w_{\theta_m}(z_{\ell-1})
\end{aligned}
\tag{11}
$$

where we have selected a *random walk* model $\big(\dot{\theta}_m(z) = w_{\theta_m}(z)\big)$ to capture the variations of the horizontal wavenumber with additive, zero-mean, Gaussian noise of covariance $R_{w_{\theta_m} w_{\theta_m}}$. The random walk model can provide constraints in the simulation,since the parameters are modeled as multivariate Gauss-Markov. The corresponding measurement model is given by

$$
p(r_s, z_\ell) = \sum_{m=1}^{M} \beta_m\big(r_s, z_s; \theta_m(z_\ell)\big)\phi_m(z_\ell) + v(z_\ell);\; \ell = 1, \cdots, L \tag{12}
$$

with

$$
\beta_m(r_s, z_s) := q\; H_0(\theta_m(z_\ell) r_s)\; \phi_m(z_s) \tag{13}
$$

This completes the development of the discrete state-space representation of the shallow ocean acoustic (normal-mode) propagation model that is embedded as a “forward propagator” into the subsequent processors for signal enhancement.

III. MODEL-BASED OCEANIC SIGNAL PROCESSING

In this section we discuss the development of the propagator for the Hudson Canyon experiment performed in 1988 in the Atlantic with the primary goal of investigating acoustic propagation (transmission and attenuation) using continuous wave data [11], [12].

In order to construct the state-space propagator, we require the set of parameters which were obtained from the experimental measurements and processing (wavenumber spectra). The raw measured data was processed (sampled, corrected, filtered, etc.) and supplied for this investigation.

The design and development of the environmentally adaptive *PF* proceeds through the following steps: (1) preprocessing the raw experimental data; (2) solving the boundary value problem [9] to obtain initial parameter sets for each temporal frequency (e.g. wavenumbers, modal coefficients, initial conditions, etc.); (3) state-space forward propagator simulation of synthetic data for *PF* analysis/design; (4) application to measured Hudson Canyon data; and (5) *PF* performance analysis.

Pre-processing of the measured pressure-field data follows the usual pattern of filtering, outlier removal and Fourier transforming to obtain the complex pressure-field as a function of depth along the array. This data along with experimental conditions (frequencies, sound-speed profiles (CTD measurements), boundary conditions, horizontal wavenumber estimators (see [12] for details) provide the input to the normal mode *BVP* solutions (SNAP [6], KRACKEN [7], etc.) yielding the output parameters. These parameters are then used as input to the state-space forward propagator [27].

The state-space propagator is then used to develop a set of synthetic pressure-field data with higher resolution than the original raw data, that is, a 46-element array at half-wave inter-element spacing rather than the 23-element array used in the experiment. This set represents the “truth” data that can be investigated when “tuning” the *PF* (e.g. number of particles, covariances, etc.). Once tuned, the processors are applied directly to the measured Hudson Canyon pressure-field data (23-elements) after re-adjusting some of the processor parameters (covariances). Here the metrics are estimated and processor performance analyzed. Since each run of the *PF* is a random realization, that is, the process noise inputs are random, an ensemble of results are estimated with ensemble statistics presented. In this way, we can achieve a detailed analysis of the processor performance prior to fielding and operational version. In this paper we constrain our discussion results to processing Hudson Canyon pressure-field measurements using a 23-element array.

IV. RESULTS

First we investigate the enhancement capabilities of the *PF* in estimating the pressure-field over a 100-member ensemble shown in Fig. 1. Using 1500-particles, we see the raw hydrophone data (dashed blue line) from the experiment as well as both maximum a-posteriori (*MAP*) estimates (red circles) and conditional mean (*CM*) estimates (dotted magenta line with circles). Both estimators appear to track the field quite well (true (mean) solution in green dashes). The corresponding innovations (residual) sequence is also shown (black). Classically, both estimators produced satisfactory zero-mean/statistical whiteness tests as well as the *WSSR* tests indicating a “tuned” processor [18].

Ensemble mode tracking results are shown in Figs. 2 and 3 for each of the modal function estimators, the *PF* (*MAP*/*CM*) and the *UKF*. In Fig. 2 we observe that the performance of the *PF* (*MAP*/*CM*) appears to track the modes quite well and better than the *UKF*. The *PF* estimators perform equivalently. Two of the modal function estimates (first two) exhibit the largest errors while the final three functional estimates are much better. The root-mean-squared (modal tracking) error for each mode is quite reasonable on the order of 10^{-5} again confirming their performance. It is interesting to note that the wavenumber estimates are constantly being adapted (adjusted) by the processor throughout the runs attesting to the nonstationary nature of the ocean statistics. The ensemble average wavenumber estimates are very reasonable: (0.206, 0.197, 0.181, 0.173, 0.142; (TRUE) 0.208, 0.199, 0.183, 0.175, 0.142. The *PF* and *CM* ensemble estimates are very close to the true values adapting to the changing ocean environment yet still preserving wavenumber values on the average. On a single realization, all three of three of the processors were capable

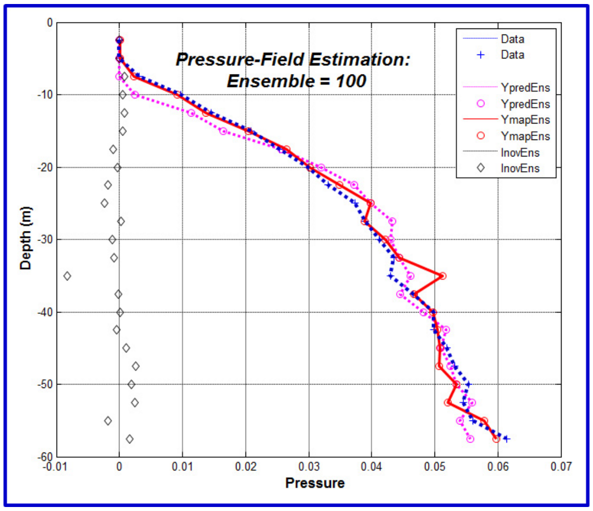

Fig. 1. Raw pressure-field data/enhanced data (blue dots) from the Hudson Canyon experiment with a 23-element hydrophone vertical array using particle filter estimators: *MAP* (red), conditional mean (*CM*) in magenta and the corresponding innovations (residuals) sequence (black diamonds).

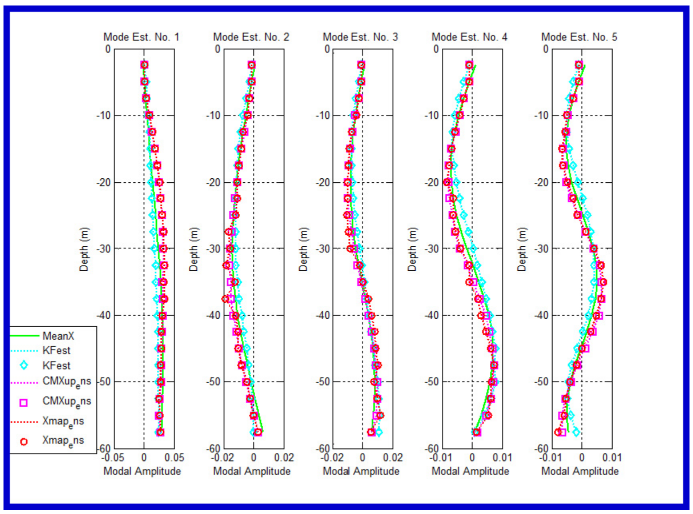

Fig. 2. Modal function tracking (estimation): Hudson Canyon data of a 23-element array (blue plus), *UKF* (turquoise dots), *MAP* (red circles) and *CM* (magenta squares) particle filters.

of predicting the correct values, but the ensemble results give a better overall performance metric.

We also illustrate the multimodal aspect of the oceanic data by observing the modal function posterior probability *PDF* estimates for mode 5 illustrated in Fig. 4. It is clear from the plots that for each depth multiple peaks appear in the posterior estimates. The wavenumber *PDF* estimate corresponding to corresponding to mode 5 is shown in Fig. 5. Again we note the multiple, well-defined peaks in the posterior distribution leading to the *MAP* parameter estimate.

The pressure-field posterior peaks over the span of the water column. Visualizing a peak at each depth produces a "smooth" estimate (*MAP*) as shown in Fig. 6. This completes the analysis of the synthesized Hudson Canyon experiment

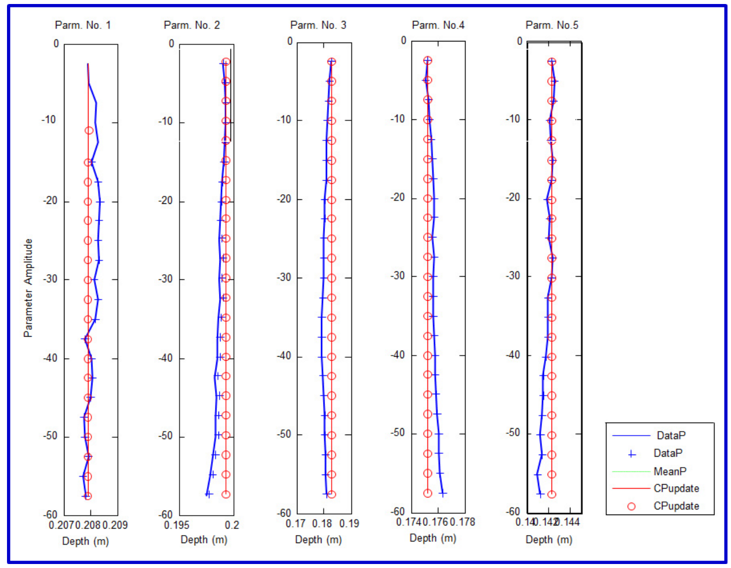

Fig. 3. Adaptive wavenumber parameter estimates from the Hudson Canyon 23-element array data using the *MAP* (red) particle filter.

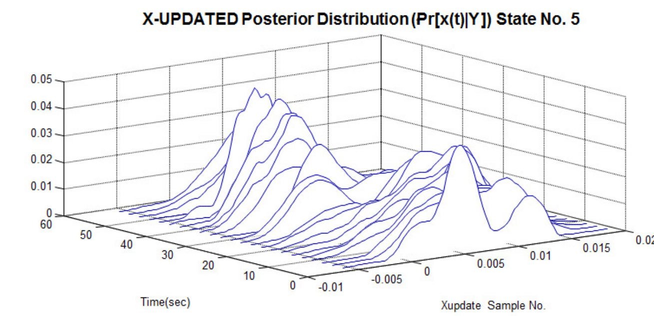

Fig. 4. *PMF* posterior estimation (mode 5) surface for Hudson Canyon 23-element array data (particle vs. time vs. probability).

and the *PF* processing performance.

V. SUMMARY

This paper has discussed the development of an environmentally adaptive processor capable of tracking modes and enhancing the raw pressure-field measurements obtained from a vertical hydrophone array in shallow water. The parametric adaption was based on simultaneously estimating the horizontal wavenumbers along with the modes and pressure-field as compared to previous work that concentrated on estimating the modal coefficients as the environmental parameters of interest [25], [26], [27]. These parameters were more challenging from a processor design perspective because of their increased sensitivity to environmental change compared to the modal coefficients. We chose a Bayesian sequential design because of the varying nature of the shallow ocean and applied a normal-mode model in state-space form to create a forward propagator. The algorithms applied were the unscented Kalman filter and the particle filter both modern approaches applied to this problem. We compared their performance and found slightly better results of the *PF* over a 100-member ensemble. Our

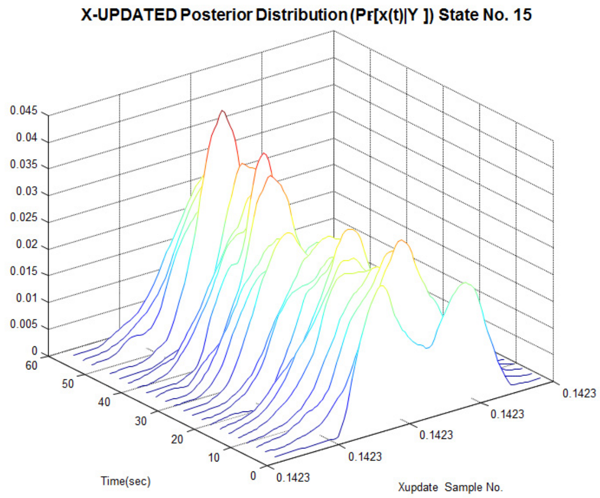

Fig. 5. *PMF* posterior estimation (wavenumber 5) surface for Hudson Canyon 23-element array data (particle vs. time vs. probability).

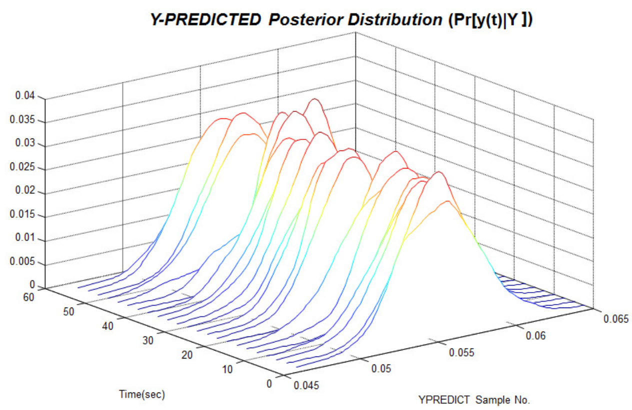

Fig. 6. Pressure-field posterior *PMF* estimation surface for Hudson Canyon data (particle vs. time vs. probability).

future efforts will be focused on extending the processors to actual measurement data.

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